

GCSE MATHS

Circle theorems

Worked Solutions

Q1

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$$\angle ACB = 124/2 = 62^\circ$$

Angle at the circumference is half the angle at the centre on the same arc.

Q2

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$$\angle AOB = 2(37) = 74^\circ$$

Angle at the centre is twice the angle at the circumference.

Q3

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$$\angle ACB = 90^\circ$$

Angle in a semicircle.

$$\angle ABC = 180 - 90 - 32 = 58^\circ$$

Angles in a triangle.

Q4

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$$\angle ACB = 248/2 = 124^\circ$$

Use the centre angle on the arc opposite C .

Q5

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$$\angle ACB = 90^\circ$$

Angle in a semicircle.

$$AB = \sqrt{6^2 + 8^2} = 10 \text{ cm}$$

Pythagoras gives the diameter.

$$r = 10/2 = 5 \text{ cm}$$

Halve the diameter.

Q6

(a) $5x + 8 = 2(2x + 11)$

Centre angle is twice circumference angle.

$$x = 14$$

Expand and rearrange.

(b) $\angle ACB = 2(14) + 11 = 39^\circ$

Substitute.

Q7

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$$\angle BCD = 180 - 108 = 72^\circ$$

Opposite cyclic angles sum to 180° .**Q8**

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$$\angle ADB = \angle ACB = 43^\circ$$

Angles in the same segment.

Q9

★★

$$5x + 30 = 180$$

Add opposite angles.

$$x = 30$$

Subtract 30 and divide by five.

$$\angle ABC = 100^\circ$$

Substitute into both expressions.

$$\angle ADC = 80^\circ$$

Q10

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$$\angle ABC = 180 - 76 = 104^\circ$$

Interior and exterior angles form a straight line.

$$\angle ADC = 180 - 104 = 76^\circ$$

Opposite cyclic angles sum to 180° .**Q11**

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$$4k + 5k = 180$$

Opposite angles sum to 180° .

$$k = 20$$

Divide by nine.

$$4k = 80^\circ$$

Find both shares.

$$5k = 100^\circ$$

Q12

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$$\angle ACB = \angle BAT$$

The angle between tangent AT and chord AB equals the angle in the alternate segment.

$$\angle ACB = 47^\circ$$

Both angles stand on chord AB .

Q13

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$$\angle OAP = \angle OBP = 90^\circ$$

Radius meets tangent at right angles.

$$\angle AOB = 360 - 90 - 90 - 54 = 126^\circ$$

Angle sum of quadrilateral $OAPB$.

Q14

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$$3x + 1 = 5x - 7$$

Tangents from P are equal.

$$x = 4$$

Rearrange.

$$PA = 3(4) + 1 = 13 \text{ cm}$$

Substitute.

Q15

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$$\angle OAP = 90^\circ$$

Radius is perpendicular to the tangent.

$$PA = \sqrt{13^2 - 5^2} = 12 \text{ cm}$$

Use Pythagoras in OAP .

Q16

$$AM = \sqrt{13^2 - 5^2} = 12 \text{ cm}$$

Use Pythagoras in OMA .

$$AB = 2AM = 24 \text{ cm}$$

A perpendicular from the centre bisects a chord.

Q17

$$OA = OB$$

Radii and tangents are equal.

$$PA = PB$$

$$\triangle OAP \cong \triangle OBP$$

SSS, with OP shared.

$$\angle OPA = 68/2 = 34^\circ$$

Corresponding angles are equal, so OP bisects APB .

Q18

$$OA = \sqrt{10^2 - 8^2} = 6 \text{ cm}$$

The tangent gives a right triangle.

$$A = 2 \left(\frac{1}{2} \times 6 \times 8 \right) = 48 \text{ cm}^2$$

Add the two congruent right triangles.

Q19

(a) $\angle OAB = \frac{180 - 136}{2} = 22^\circ$

$OA = OB$, so OAB is isosceles.

(b) $\angle ACB = 136/2 = 68^\circ$

Angle at the circumference.

Q20

$$\angle ACB = 90^\circ$$

Angle in a semicircle.

$$x + 2x = 90 \Rightarrow x = 30$$

The other angles sum to 90° .

$$\angle ABC = 60^\circ$$

Double x .

Q21

$$(4x + 8) + (6x + 12) = 180$$

The two other quadrilateral angles are right angles.

$$10x + 20 = 180 \Rightarrow x = 16$$

Collect and solve.

Q22

$$\angle BAD = 180 - 112 = 68^\circ$$

$$\angle ABD = \angle BDA$$

$$\angle ABD = (180 - 68)/2 = 56^\circ$$

Opposite cyclic angles.

$AB = AD$ makes ABD isosceles.

Use the triangle angle sum.

Q23

$$\sin \angle OPA = \frac{OA}{OP} = \frac{1}{2}$$

$$\angle OPA = 30^\circ$$

$$\angle APB = 2(30) = 60^\circ$$

OAP is right-angled at A .

The acute angle with sine one half is 30° .

Congruent tangent triangles make OP the angle bisector.

Q24

$$2x + 2(x + 10) = 180$$

$$4x = 160 \Rightarrow x = 40$$

$$\angle ACB = \frac{2x}{2} = 40^\circ$$

Equal radii make the base angles equal.

Solve the triangle angle equation.

Apply the centre-circumference theorem.