

GCSE MATHS

Circle theorems

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Centre and circumference

The angle at the centre is twice the angle at the circumference standing on the same arc. Check whether the centre angle is reflex.

Diameter and equal segments

An angle in a semicircle is 90° . Angles standing on the same chord in the same segment are equal.

Cyclic quadrilaterals

Opposite angles of a quadrilateral with all four vertices on one circle sum to 180° .

Tangents and chords

A tangent is perpendicular to the radius at contact. Tangents from the same external point have equal lengths. The angle between a tangent and a chord equals the angle in the alternate segment. A perpendicular from the centre bisects a chord.

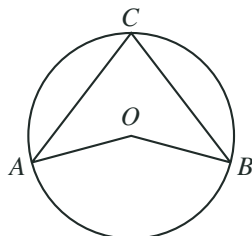
Calculate from the given information; do not measure the diagrams.

1. Centre, circumference and diameters

Example Problem 1

**

O is the centre and the smaller angle $AOB = 150^\circ$. Find angle ACB .



The idea

1. Both angles stand on arc AB opposite C .
2. Use the centre-to-circumference factor of two.

Worked Solution

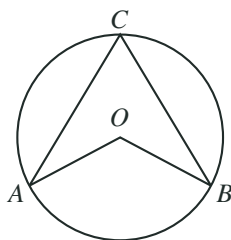
$$\angle ACB = 150/2 = 75^\circ$$

The circumference angle is half the centre angle on the same arc.

1 Centre, circumference and diameters

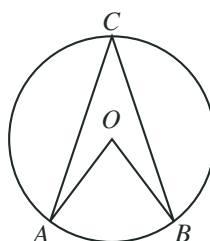
Q1 O is the centre. The smaller angle AOB is 124° . Find angle ACB .

*



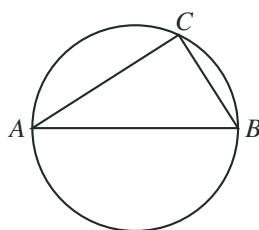
Q2 O is the centre and $ACB = 37^\circ$. Find the smaller angle AOB .

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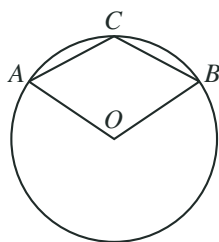


Q3 AB is a diameter. Angle $CAB = 32^\circ$. Find angle ABC .

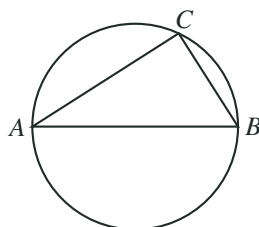
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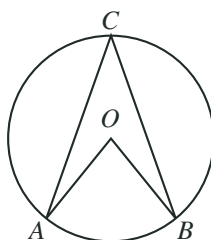
- Q4** O is the centre. The reflex angle AOB is 248° and C lies on the minor arc AB . Find angle ACB . **



- Q5** AB is a diameter. $AC = 6$ cm and $BC = 8$ cm. Find the circle's radius. **



- Q6** O is the centre. The smaller angle $AOB = (5x + 8)^\circ$ and $ACB = (2x + 11)^\circ$. (a) Find x . (b) Find angle ACB . ***

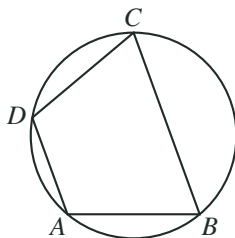


2. Cyclic quadrilaterals and equal angles

Example Problem 2

**

$ABCD$ is cyclic. $ABC = (2x + 15)^\circ$ and $ADC = (3x + 5)^\circ$. Find x .



The idea

1. Identify opposite angles from the vertex order.
2. Form one equation using their sum.

Worked Solution

$$2x + 15 + 3x + 5 = 180$$

$$5x = 160 \Rightarrow x = 32$$

Opposite cyclic angles add to 180° .

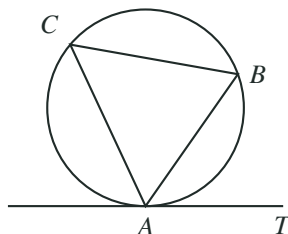
Collect terms and solve.

2. Cyclic quadrilaterals and equal angles

Example Problem 3

**

AT is tangent at A . The angle BAT is 55° . Find angle ACB .



The idea

1. Identify the chord AB that forms the angle with the tangent.
2. Use the angle in the opposite segment standing on that chord.

Worked Solution

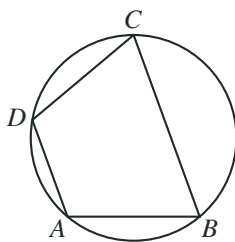
$$\angle ACB = \angle BAT = 55^\circ$$

Apply the alternate segment theorem.

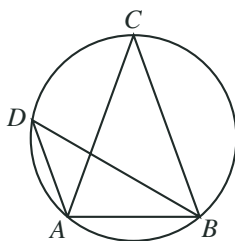
2 Cyclic quadrilaterals and equal angles

Q7 $ABCD$ is cyclic. Angle $DAB = 108^\circ$. Find angle BCD .

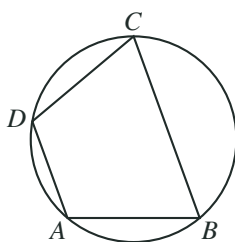
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**Q8** C and D lie in the same segment of chord AB . Angle $ACB = 43^\circ$. Find angle ADB .

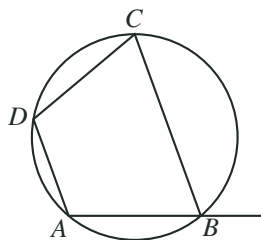
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**Q9** $ABCD$ is cyclic. Angle $ABC = (3x + 10)^\circ$ and $ADC = (2x + 20)^\circ$. Find both angles.

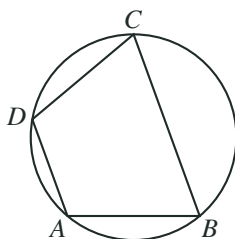
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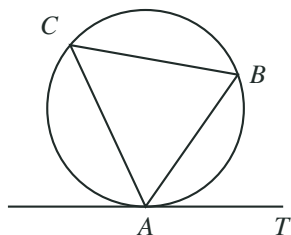
- Q10** $ABCD$ is cyclic. The exterior angle formed by extending AB beyond B is 76° . Find angle ADC . **



- Q11** $ABCD$ is cyclic. Angles DAB and BCD are in the ratio $4 : 5$. Find them. ***



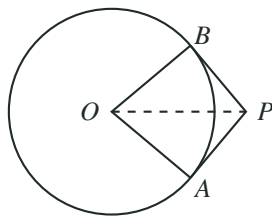
- Q12** AT is tangent at A and angle $BAT = 47^\circ$. Find angle ACB , giving a reason. **



3. Tangents and chords

Example Problem 4

PA and PB are tangents. O is the centre, $OA = 3$ cm and $OP = 5$ cm. Find PA .



The idea

1. Join the centre to the tangent contact point.
2. Use the resulting right triangle.

Worked Solution

$$\angle OAP = 90^\circ$$

Radius is perpendicular to tangent.

$$PA^2 = 5^2 - 3^2 = 16$$

Use Pythagoras.

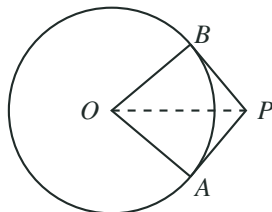
$$PA = 4 \text{ cm}$$

Take the positive length.

3 Tangents and chords

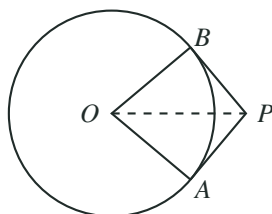
Q13 PA and PB are tangents and O is the centre. Angle $APB = 54^\circ$. Find the smaller angle AOB .

**



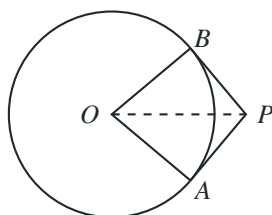
Q14 PA and PB are tangents. $PA = (3x + 1)$ cm and $PB = (5x - 7)$ cm. Find their length.

**

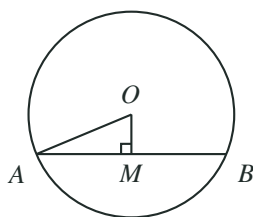


Q15 PA is tangent at A . O is the centre, $OA = 5$ cm and $OP = 13$ cm. Find PA .

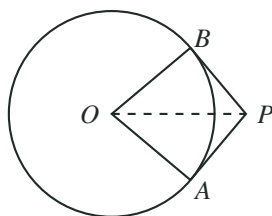
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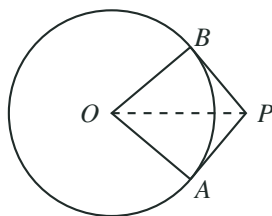
- Q16** O is the centre and OM is perpendicular to chord AB . The radius is 13 cm and $OM = 5$ cm. Find AB . ***



- Q17** PA and PB are tangents, and O is the centre. Angle $APB = 68^\circ$. Find angle OPA . ***



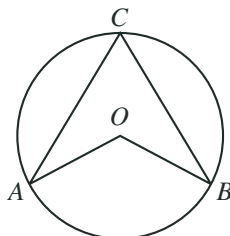
- Q18** PA and PB are tangents. O is the centre. If $OP = 10$ cm and $PA = 8$ cm, find the area of quadrilateral $OAPB$. ***



4. Combining circle theorems

Example Problem 5

O is the centre and $OAB = 28^\circ$. Find angle ACB .



The idea

1. Use equal radii to find the centre angle.
2. Then connect that angle to the circumference.

Worked Solution

$$\angle AOB = 180 - 2(28) = 124^\circ$$

$$\angle ACB = 124/2 = 62^\circ$$

$OA = OB$ gives equal base angles.

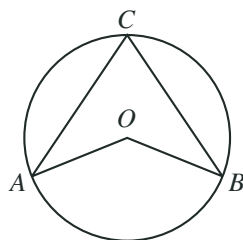
The circumference angle is half the centre angle.

4 Combining circle theorems

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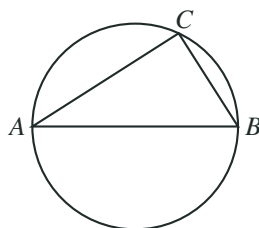
Q19 O is the centre and the smaller angle $AOB = 136^\circ$. Find (a) OAB ; (b) ACB .

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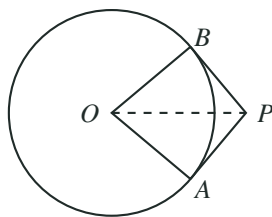


Q20 AB is a diameter. Angle ABC is twice CAB . Find both angles.

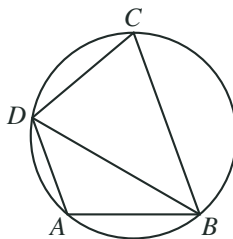
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- Q21** PA and PB are tangents, O is the centre, and $APB = (4x + 8)^\circ$. The smaller angle $AOB = (6x + 12)^\circ$. Find x .

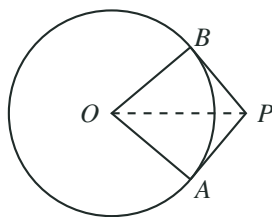


- Q22** $ABCD$ is cyclic. $AB = AD$ and angle $BCD = 112^\circ$. Find angle ABD .



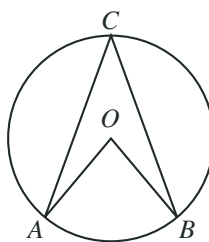
Q23 PA and PB are tangents. O is the centre and $OP = 2OA$. Find angle APB . Give reasons for your answer.

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Q24 O is the centre. The smaller angle AOB is $2x^\circ$ and $OAB = (x + 10)^\circ$. Find angle ACB .

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Solving principles

1. Identify the arc

Match the two endpoints of the chord before comparing angles.

2. Name the reason

Give the relevant theorem beside each angle you calculate.

3. Add useful radii

Equal radii create isosceles triangles. A radius to a tangent creates a right angle.

4. Use the angles already found

Use triangle angle sums and straight lines to connect the circle theorems.