

GCSE MATHS

Circle equations and tangents

Worked Solutions

Q1

★

$$r = \sqrt{49} = 7$$

Take the positive square root.

Q2

★

$$r^2 = 9^2 = 81$$

Square the radius.

$$x^2 + y^2 = 81$$

Use the circle equation.

Q3

★★

$$r = 12/2 = 6$$

Halve the diameter.

$$x^2 + y^2 = 6^2 = 36$$

Square the radius.

Q4

★★

$$r^2 = 6^2 + 8^2 = 100$$

Use Pythagoras.

$$r = 10$$

Take the positive square root.

Q5

★★

$$\pi r^2 = 36\pi \Rightarrow r^2 = 36$$

Cancel π .

$$x^2 + y^2 = 36$$

Use the squared radius.

Q6

★★★

$$r^2 = 4^2 + 4^2 = 32$$

Use one vertex; symmetry gives the other three.

$$x^2 + y^2 = 32$$

Use the circle equation with centre at the origin.

Q7

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$$x^2 = 25 - 16 = 9 \Rightarrow x = \pm 3$$

Keep both signs of the square root.

Q8

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$$y^2 = 169 - 144 = 25 \Rightarrow y = \pm 5$$

Subtract the square of x .**Q9**

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(a) $2^2 + 4^2 = 20 < 25$

Its distance from the origin is less than the radius.

(b) $4^2 + 4^2 = 32 > 25$

Its distance from the origin is greater than the radius.

Q10

$$2x^2 = 18 \Rightarrow x = \pm 3$$

Substitute $y = x$.

$$(-3, -3)$$

Pair each x -value with its corresponding y -value.

$$(3, 3)$$

Q11

$$x^2 + 4x^2 = 45 \Rightarrow x = \pm 3$$

Substitute and solve.

$$(-3, 6)$$

Pair each x -value with its corresponding y -value.

$$(3, -6)$$

Q12

$$m_r = 4/3$$

Find the radius gradient.

$$m_t = -3/4$$

Take the negative reciprocal.

Q13

$$m_t = -4/3$$

Take the negative reciprocal of the radius gradient.

$$y - 3 = -\frac{4}{3}(x - 4)$$

Use the contact point.

$$4x + 3y = 25$$

Rearrange.

Q14

**

$$y = 7$$

A vertical radius has a horizontal tangent.

Q15

**

$$x = -6$$

A horizontal radius has a vertical tangent.

Q16

$$m_t = -5/12$$

The radius gradient is 12/5.

$$y - 12 = -\frac{5}{12}(x - 5)$$

Use the point-gradient equation.

$$5x + 12y = 169$$

Multiply through and rearrange.

Q17

$$6x + 8y = 100$$

Find the equation of the tangent using the perpendicular radius.

$$y = 0 \Rightarrow x = 100/6 = 50/3$$

Set y to zero for the x -intercept.

Q18

$$x^2 = 25 - 9 = 16 \Rightarrow x = \pm 4$$

Find both endpoints.

$$4 - (-4) = 8$$

Subtract their x -coordinates.

Q19

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$$3x + 4y = 25$$

$$A = (25/3, 0)$$

$$B = (0, 25/4)$$

$$\text{Area} = \frac{1}{2} \times \frac{25}{3} \times \frac{25}{4} = \frac{625}{24}$$

Find the equation of the tangent equation.

Set y and x to zero in turn.

Use perpendicular base and height.

Q20

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$$3x + 4y = 25$$

$$3x - 4y = 25$$

$$6x = 50 \Rightarrow x = 25/3$$

$$y = 0$$

Write both tangent equations.

Add the equations.

Substitute back.

Q21

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$$x^2 + (x + 1)^2 = 25$$

$$x^2 + x - 12 = 0$$

$$(x + 4)(x - 3) = 0 \Rightarrow x = -4, 3$$

$$(-4, -3)$$

$$(3, 4)$$

Substitute the line into the circle.

Expand, subtract 25 and divide by two.

Factorise.

Calculate y .

Q22

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$$t^2 + 4t^2 = 80 \Rightarrow t^2 = 16$$

$$t = \pm 4$$

Substitute the coordinates.

Keep both roots.

Q23

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$$6x + 8y = 100 \Rightarrow T = (50/3, 0)$$

$$Q = (6, -8)$$

$$6x - 8y = 100$$

Find the first tangent and its intercept.

Reflection in the x -axis leaves T and the circle unchanged, so it maps one tangent to the other.

This reflected tangent passes through T and Q .