

GCSE MATHS

Circle equations and tangents

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Circle equation

A circle centred at the origin with radius r has equation $x^2 + y^2 = r^2$. This follows from Pythagoras.

Points

Compare $x^2 + y^2$ with r^2 : equality means on the circle, a smaller value means inside, and a larger value means outside.

Tangents

A tangent is perpendicular to the radius at its contact point. For non-vertical lines, perpendicular gradients multiply to -1 .

Horizontal and vertical tangents

At the top and bottom of the circle, the tangent is horizontal. At the left and right extremes, it is vertical.

A calculator may be used. Give exact answers.

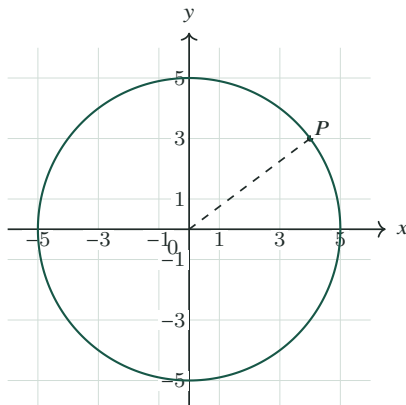
Calculate from the given coordinates; do not measure the diagrams.

1. The equation of a circle

Example Problem 1

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Write the equation of the circle with centre $(0, 0)$ passing through $(4, 3)$.



The idea

1. Calculate the squared radius with Pythagoras.

Worked Solution

$$r^2 = 4^2 + 3^2 = 25$$

$$x^2 + y^2 = 25$$

The point gives horizontal and vertical displacements.

Use the circle equation.

1 The equation of a circle

Q1 Find the radius of $x^2 + y^2 = 49$.

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Q2 Write the equation of a circle centred at the origin with radius 9.

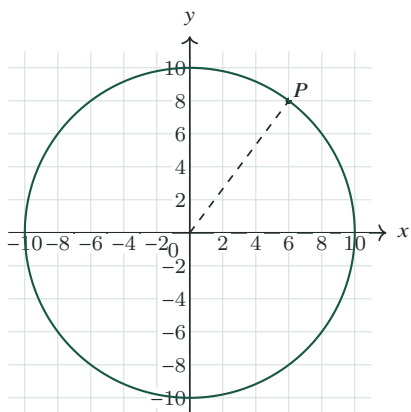
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Q3 A circle centred at the origin has diameter 12. Write its equation.

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Q4 A circle centred at the origin passes through $(6, 8)$. Find its radius.

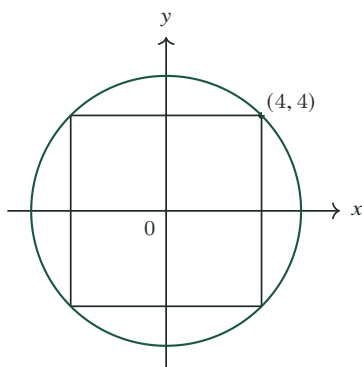
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Q5 A circle centred at the origin has area 36π . Write its equation.

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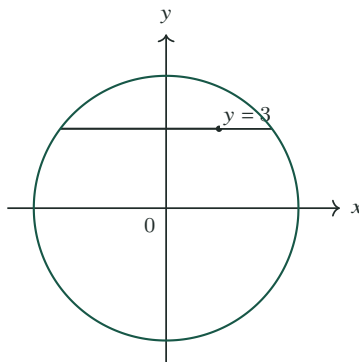
Q6 A square has vertices $(4, 4)$, $(-4, 4)$, $(-4, -4)$ and $(4, -4)$. A circle centred at the origin passes through its vertices. Write down the equation of the circle.



2. Points and intersections

Example Problem 2

Find the intersections of $x^2 + y^2 = 25$ with $y = 3$.



The idea

1. Substitute $y = 3$, then keep both square roots.

Worked Solution

$$x^2 + 9 = 25 \Rightarrow x^2 = 16$$

$$x = \pm 4$$

$$(-4, 3)$$

$$(4, 3)$$

Substitute and rearrange.

There are two possible x -values.

Pair each x with $y = 3$.

2 Points and intersections

Q7 The point $(x, 4)$ lies on the circle $x^2 + y^2 = 25$. Find the two possible values of x .

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Q8 The point $(12, y)$ lies on the circle $x^2 + y^2 = 169$. Find the two possible values of y .

**

Q9 A circle has equation $x^2 + y^2 = 25$. State whether each point is inside, on or outside the circle. Explain each answer.

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(a) $(2, 4)$

(b) $(4, 4)$

Q10 Find the intersections of $x^2 + y^2 = 18$ and $y = x$.

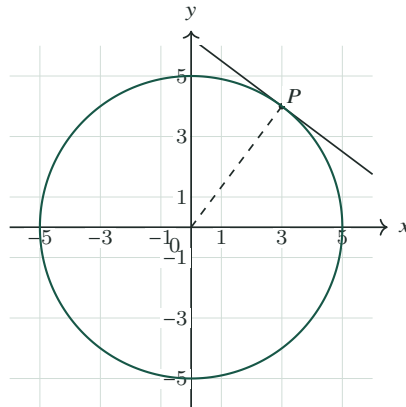
Q11 Find the intersections of $x^2 + y^2 = 45$ and $y = -2x$.

3. Equations of tangents

Example Problem 3

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Find the equation of the tangent to $x^2 + y^2 = 25$ at $(3, 4)$.



The idea

1. Find the radius gradient, take its negative reciprocal, and use the contact point.

Worked Solution

$$m_r = 4/3$$

Radius and tangent are perpendicular.

$$m_t = -3/4$$

$$y - 4 = -\frac{3}{4}(x - 3)$$

Use the line through $(3, 4)$.

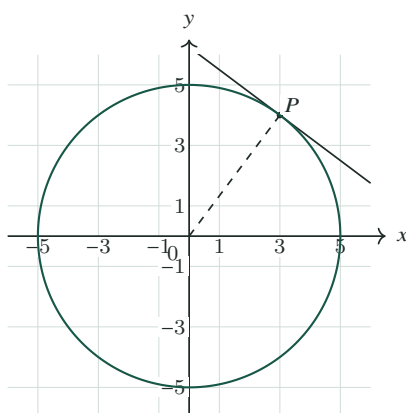
$$3x + 4y = 25$$

Multiply by four and rearrange.

3 Equations of tangents

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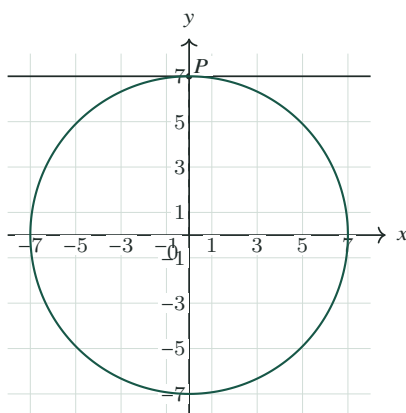
Q12 A circle has centre $(0, 0)$ and passes through $(3, 4)$. Find the gradient of the tangent at that point.



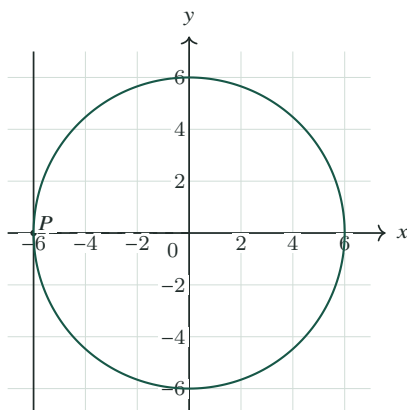
Q13 Find the equation of the tangent to $x^2 + y^2 = 25$ at $(4, 3)$.

Q14 Find the equation of the tangent to $x^2 + y^2 = 49$ at $(0, 7)$.

**



Q15 Find the equation of the tangent to $x^2 + y^2 = 36$ at $(-6, 0)$. **



Q16 Find the equation of the tangent to $x^2 + y^2 = 169$ at $(5, 12)$. ***

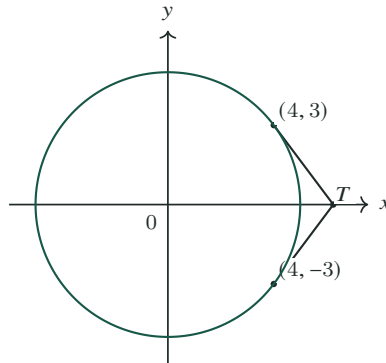
Q17 Find the x -intercept of the tangent to $x^2 + y^2 = 100$ at $(6, 8)$. ****

4. Combining circle and line geometry

Example Problem 4

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The tangents to the circle $x^2 + y^2 = 25$ at $(4, 3)$ and $(4, -3)$ meet at T . Find the coordinates of T .



The idea

1. Write both tangent equations, then solve them simultaneously.

Worked Solution

$$4x + 3y = 25$$

$$4x - 3y = 25$$

$$8x = 50 \Rightarrow x = 25/4$$

$$y = 0$$

$$(25/4, 0)$$

Use the radius and the perpendicular tangent gradients.

Add the equations.

Substitute into either equation.

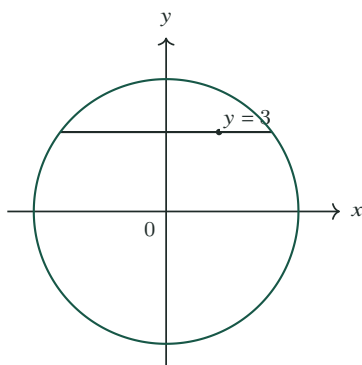
State the intersection.

4 Combining circle and line geometry

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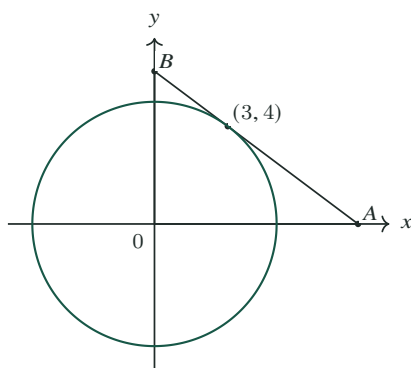
Q18 A horizontal chord of $x^2 + y^2 = 25$ lies on $y = 3$. Find its length.

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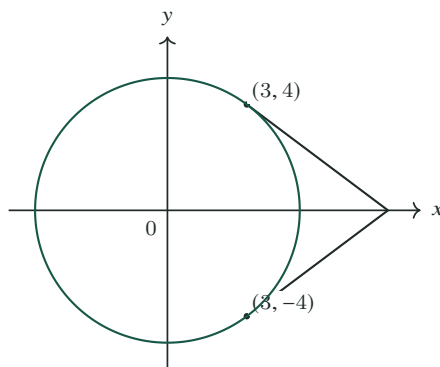
Q19 The tangent at $(3, 4)$ to $x^2 + y^2 = 25$ meets the axes at A and B . Find the area of triangle OAB , where O is the origin.

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Q20 Find where the tangents to $x^2 + y^2 = 25$ at $(3, 4)$ and $(3, -4)$ intersect.

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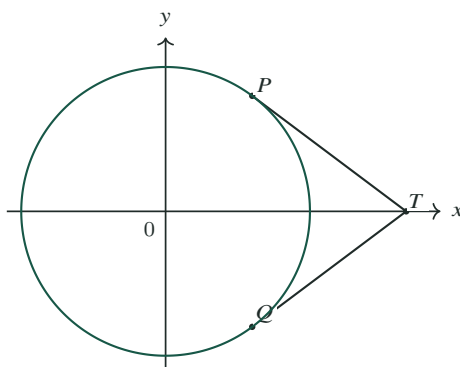


Q21 Find the intersections of $x^2 + y^2 = 25$ and $y = x + 1$.

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Q22 The point $(t, 2t)$ lies on the circle $x^2 + y^2 = 80$. Find the two possible values of t .

Q23 The tangent at $P = (6, 8)$ to $x^2 + y^2 = 100$ meets the positive x -axis at T . A second tangent from T touches the circle at Q . Find Q .



Solving principles

1. Use the squared radius

The constant in the circle equation is the square of the radius.

2. Keep both signs

A square-root step can give two coordinate values.

3. Start with the radius

Find its gradient before calculating a tangent gradient.

4. Check the contact point

It must satisfy both the circle and the tangent equations.