

GCSE MATHS

Bounds and error intervals

Worked Solutions

Q1

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$$h = 0.1/2$$

Find half the rounding unit.

$$7.35 \leq x < 7.45$$

Subtract and add the half-unit.

Q2

*

$$h = 10/2$$

Find half the rounding unit.

$$355 \leq x < 365$$

Subtract and add the half-unit.

Q3

*

$$h = 0.001/2$$

Find half the rounding unit.

$$0.0475 \leq x < 0.0485$$

Subtract and add the half-unit.

Q4

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$$\text{rounding unit} = 0.001$$

The second significant figure is in the thousandths column.

$$0.0315 \leq x < 0.0325$$

Use half of 0.001 on either side.

Q5

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$$l_{\text{recorded}} = \frac{12.35 + 12.45}{2} = 12.4$$

Find the midpoint.

$$12.45 - 12.35 = 0.1$$

The interval width is the rounding unit.

Q6

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$$5.5 \leq a, b < 6.5$$

Write the common error interval.

$$6.45 - 5.5 = 0.95 > 0.9$$

Choose two permissible lengths.

Q7

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$$L = 8.2 + 3.1 = 11.3$$

Use both lower bounds.

$$U = 8.4 + 3.3 = 11.7$$

Use both upper bounds.

Q8

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$$L = 8.2 - 3.3 = 4.9$$

Subtract the largest b from the smallest a.

$$U = 8.4 - 3.1 = 5.3$$

Subtract the smallest b from the largest a.

Q9

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$$l < 12.5$$

Find upper bounds.

$$w < 8.5$$

$$P < 2(12.5 + 8.5) = 42 \text{ cm}$$

Both dimensions increase the perimeter.

Q10

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$$l \geq 3.95$$

Find lower bounds.

$$w \geq 1.95$$

$$A \geq 3.95(1.95) = 7.7025 \text{ m}^2$$

Multiply positive lower bounds.

Q11

$$s < 5.05$$

Upper side bound.

$$s^2 - 25 < 5.05^2 - 25 = 0.5025$$

Square before subtracting 25.

Q12

$$19.5 \leq a < 20.5$$

Write the intervals.

$$11.5 \leq b < 12.5$$

$$7 < a - b < 9$$

Use opposite extremes.

Q13

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$$U = \frac{10.5}{1.9} = \frac{105}{19}$$

Largest numerator and smallest positive denominator.

Q14

$$d < 125$$

Use the upper distance and lower time.

$$t \geq 1.95$$

$$v_U = \frac{125}{1.95} = 64.10256 \dots$$

Divide before rounding.

$$v_U = 64.10 \text{ km/h (2 d.p.)}$$

State the rounded bound.

Q15

$$m \geq 245$$

Use lower mass and upper volume.

$$V < 30.5$$

$$\rho_L = \frac{245}{30.5} = 8.032786 \dots$$

Density is mass divided by volume.

$$\rho_L = 8.033 \text{ g/cm}^3 \text{ (3 d.p.)}$$

Round the limiting value.

Q16

$$\frac{a}{a+b} = \frac{1}{1+b/a}$$

Rewrite to expose how each measurement acts.

$$a < 6.1$$

A larger a and smaller b increase the ratio. Use the relevant endpoint when calculating the bound, even if the measurement cannot equal it.

$$b = 1.9$$

$$U = \frac{6.1}{6.1+1.9} = 0.7625$$

Use the same value of a in both places.

Q17

$$x < 10.5$$

A larger positive x makes the reciprocal smaller.

$$L = \frac{1}{10.5^2} = \frac{4}{441}$$

Square the upper bound, then take its reciprocal.

Q18

$$\frac{a-b}{a+b} = 1 - \frac{2}{a/b+1}$$

a/b is smallest as $a = 2.9$, $b < 1.1$

$$L = \frac{2.9 - 1.1}{2.9 + 1.1} = 0.45$$

Divide the numerator and denominator of the second fraction by b .

A smaller a/b makes the subtracted fraction larger. Use the relevant endpoint when calculating the bound, even if the measurement cannot equal it.

Use each measurement consistently.

Q19

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$$7.241 \rightarrow 7.24$$

$$7.248 \rightarrow 7.25$$

$$7.241 \rightarrow 7.2$$

$$7.249 \rightarrow 7.2$$

Two allowed values disagree at 2 decimal places.

All values agree at 1 decimal place.

Q20

$$3.4561 \rightarrow 3.456$$

$$3.4564 \rightarrow 3.456$$

$$3.4561 \neq 3.4563$$

Both limits round identically at 3 decimal places.

Allowed values differ at 4 decimal places.

Q21

$$49.5 \leq L < 50.5$$

$$9.9 \leq L/5 < 10.1$$

Bound the original rod.

Divide both bounds by the exact count of five.

Q22

$$P = 2(l + l/4) = \frac{5}{2}l$$

$$l_U = \frac{40.5}{2.5} = 16.2$$

$$w_U = 4.05$$

$$A_U = 16.2(4.05) = 65.61 \text{ cm}^2$$

Use the exact relation between dimensions.

Find both consistent upper dimensions.

Multiply.

Q23

★★★★

$$3.5 \leq x < 4.5$$

Write the interval.

$$-0.5 \leq x - 4 < 0.5$$

Subtract four.

$$0 \leq (x - 4)^2 \leq 0.25$$

Zero is inside the interval; the lower endpoint gives the maximum 0.25.

Q24

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$$7.5 \leq l < 8.5$$

The width depends on the length.

$$w = 12 - l$$

$$A = l(12 - l) = 36 - (l - 6)^2$$

Complete the square.

$$A_U = 36 - 1.5^2 = 33.75$$

On this interval, the smallest l is closest to 6.

$$A_L = 36 - 2.5^2 = 29.75$$

The largest limiting l gives the smallest limiting area.