

GCSE MATHS

Bounds and error intervals

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

Contents

	Questions	Difficulty	Page
Topic recap			2
1. Rounding and error intervals	Q1–6	**	3
2. Bounds of sums and products	Q7–12	***	5
3. Bounds of quotients and rates	Q13–18	****	7
4. Accuracy and connected measurements	Q19–24	****	10
Solving principles			14

Topic recap

Half the rounding unit

A length given as 8.2 cm to the nearest 0.1 cm has $8.15 \leq l < 8.25$. The half-unit is 0.05, not 0.1.

Combining positive quantities

For positive measurements, add lower bounds for a lower sum; multiply lower bounds for a lower product. For $a - b$, its lower bound uses the smallest a and largest b .

Division

To make a/b as large as possible, increase the positive numerator and decrease the positive denominator.

Justified accuracy

Round both limiting values to the proposed accuracy. Every possible value must round to the same result.

A calculator may be used.

1. Rounding and error intervals

Example Problem 1

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A mass is 2.6 kg to the nearest 0.1 kg. Write its error interval.

The idea

1. Identify the place to which the measurement was rounded.
2. Use half that unit on each side of the recorded value.

Worked Solution

$$h = 0.1/2 = 0.05$$

Half the rounding unit.

$$2.55 \leq m < 2.65$$

Subtract and add 0.05.

1 Rounding and error intervals

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Q1 A length is 7.4 cm to the nearest 0.1 cm. Write the error interval for its value x , measured in cm.

*

Q2 A mass is 360 g to the nearest 10 g. Write the error interval for its value x , measured in g.

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Q3 A mass is 0.048 kg to the nearest 0.001 kg. Write the error interval for its value x , measured in kg.

*

Q4 A positive number is 0.032 to two significant figures. Write its error interval.

**

Q5 A length has $12.35 \leq l < 12.45$ cm. State the recorded length and its rounding accuracy.

**

Q6 Two lengths both round to 6 cm to the nearest centimetre. Can their difference exceed 0.9 cm? Give an example.

**

2. Bounds of sums and products

Example Problem 2

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A rectangle is 6 cm by 3 cm, each to the nearest centimetre. Find the upper bound of its area.

The idea

1. Bound each measured dimension.
2. Use upper bounds for both positive factors.

Worked Solution

$$l < 6.5$$

$$w < 3.5$$

$$A_U = 6.5(3.5) = 22.75 \text{ cm}^2$$

Find upper bounds.

Both positive dimensions increase area.

2 Bounds of sums and products

Q7 $8.2 \leq a < 8.4$ and $3.1 \leq b < 3.3$. Find the lower and upper bounds of $a + b$.

**

Q8 $8.2 \leq a < 8.4$ and $3.1 \leq b < 3.3$. Find the lower and upper bounds of $a - b$.

**

Q9 A rectangle is 12 cm by 8 cm, both to the nearest centimetre. Find the upper bound of its perimeter.

**

Q10 A rectangle is 4.0 m by 2.0 m, both to the nearest 0.1 m. Find the lower bound of its area.

**

Q11 A square has side 5.0 cm to the nearest 0.1 cm. Find the upper bound of the difference between its area and 25 cm^2 .

Q12 Two lengths are recorded as 20 cm and 12 cm, each to the nearest centimetre. Find the lower and upper bounds of the longer length minus the shorter length.

3. Bounds of quotients and rates

Example Problem 3

$4.9 \leq a < 5.1$ and $1.9 \leq b < 2.1$. Find the upper bound of $a/(a+b)$.

The idea

1. The two appearances of a are the same measurement.
2. Minimise b/a to maximise the whole fraction.

Worked Solution

$$\frac{a}{a+b} = \frac{1}{1+b/a}$$
$$U = \frac{5.1}{5.1+1.9} = \frac{51}{70}$$

Rewrite using the ratio b/a .

Use large a and small b throughout.

3 Bounds of quotients and rates

Q13 $9.5 \leq a < 10.5$ and $1.9 \leq b < 2.1$. Find the upper bound of a/b .

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Q14 A journey is 120 km to the nearest 10 km and takes 2.0 hours to the nearest 0.1 hour. Find the upper bound of average speed, to 2 decimal places.

Q15 A block has mass 250 g to the nearest 10 g and volume 30 cm^3 to the nearest 1 cm^3 . Find the lower bound of density, to 3 decimal places.

Q16 $5.9 \leq a < 6.1$ and $1.9 \leq b < 2.1$. Find the upper bound of $a/(a+b)$.

Q17 A positive number x is 10 to the nearest whole number. Find the lower bound of $1/x^2$.

Q18 $2.9 \leq a < 3.1$ and $0.9 \leq b < 1.1$. Find the lower bound of $(a-b)/(a+b)$.

4. Accuracy and connected measurements

Example Problem 4

$12.361 \leq x < 12.364$. Give x to the greatest justified number of decimal places.

The idea

1. Try rounding both limits to the same accuracy.
2. Check that a finer accuracy would give different results.

Worked Solution

$$12.361 \rightarrow 12.36$$

Both limits agree at 2 decimal places.

$$12.364 \rightarrow 12.36$$

$$12.361 \neq 12.363$$

Allowed values disagree at 3 decimal places.

$$x = 12.36 \text{ to 2 d.p.}$$

State only the justified accuracy.

4. Accuracy and connected measurements

Example Problem 5

★★★★

A positive number x is 3 to the nearest whole number. Find the lower and upper bounds of $7 - (x - 3)^2$.

The idea

1. Locate where the squared expression is zero before testing the endpoints.
2. Subtracting a square reverses which value gives the larger result.

Worked Solution

$$2.5 \leq x < 3.5$$

Write the error interval.

$$-0.5 \leq x - 3 < 0.5$$

Subtract 3 throughout.

$$0 \leq (x - 3)^2 \leq 0.25$$

The square is zero at $x = 3$ and reaches 0.25 at $x = 2.5$.

$$6.75 \leq 7 - (x - 3)^2 \leq 7$$

Subtract from 7; both bounds are attained.

4 Accuracy and connected measurements

★★★★

Q19 $7.241 \leq x < 7.249$. Give x to the greatest number of decimal places that is justified for every value in the interval.

★★

Q20 $3.4561 \leq x < 3.4564$. Give x to the greatest justified number of decimal places.

★★★

Q21 A rod is 50 cm to the nearest centimetre. It is cut into five exactly equal pieces. Find the error interval for one piece, p .

★★★

Q22 A rectangle has perimeter 40 cm to the nearest centimetre. Its width is exactly one quarter of its length. Find the upper bound of its area.

★★★

Q23 A positive number x is 4 to the nearest whole number. Find the lower and upper bounds of $(x - 4)^2$. Explain why simply squaring both endpoints would fail.

Q24 A rectangle has exact perimeter 24 cm. Its length is 8 cm to the nearest centimetre. Find the lower and upper bounds of its area.

Solving principles

1. Identify the rounding unit

Write an interval for every measured quantity before calculating.

2. Choose extremes from the expression

For $a - b$, use a smaller a and larger b for the lower bound. For positive a/b , use a larger a and smaller b for the upper bound.

3. Use the same measurement consistently

If a measurement occurs twice, it has one value throughout. Simplify an expression before choosing bounds.

4. Keep unrounded values

Use unrounded bounds in later calculations. Round only the final requested answer.