

GCSE MATHS

Arcs and sectors

Worked Solutions

Q1

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$$C = 2\pi(7) = 14\pi \text{ cm}$$

Use twice the radius.

Q2

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$$r = 10/2 = 5 \text{ cm}$$

Halve the diameter.

$$A = \pi(5)^2 = 25\pi \text{ cm}^2$$

Square the radius.

Q3

★★

$$2\pi r = 18\pi \Rightarrow r = 9$$

Find the radius.

$$A = 81\pi \text{ cm}^2$$

Square it and multiply by π .**Q4**

★★

$$r^2 = 49 \Rightarrow r = 7$$

Take the positive radius.

$$C = 2\pi(7) = 14\pi \text{ cm}$$

Use the circumference formula.

Q5

★★

(a) $C_1 : C_2 = 3 : 5$

Circumference is proportional to radius.

(b) $A_1 : A_2 = 3^2 : 5^2 = 9 : 25$

Area is proportional to radius squared.

Q6

★★

$$C = 70\pi \text{ cm}$$

Distance per revolution.

$$d = 100(70\pi) = 7000\pi \text{ cm}$$

Multiply by the exact revolution count.

$$d = 70\pi \text{ m}$$

Divide by 100 to convert centimetres to metres.

Q7

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$$L = \frac{60}{360} 2\pi(6)$$

$$L = 2\pi \text{ cm}$$

The sector is $\frac{1}{6}$ of the circle, so multiply the full circumference by $\frac{1}{6}$.

Simplify.

Q8

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$$L = \frac{80}{360} 2\pi(9)$$

$$L = 4\pi \text{ cm}$$

The sector is $\frac{2}{9}$ of the circle, so multiply the full circumference by $\frac{2}{9}$.

Simplify.

Q9

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$$L = \frac{150}{360} 2\pi(12)$$

$$L = 10\pi \text{ cm}$$

The sector is $\frac{5}{12}$ of the circle, so multiply the full circumference by $\frac{5}{12}$.

Simplify.

Q10

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$$A = \frac{90}{360} \pi(8)^2$$

$$A = 16\pi \text{ cm}^2$$

The sector is $\frac{1}{4}$ of the circle, so multiply the full area by $\frac{1}{4}$.

Simplify.

Q11

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$$A = \frac{120}{360} \pi(12)^2$$

$$A = 48\pi \text{ cm}^2$$

The sector is $\frac{1}{3}$ of the circle, so multiply the full area by $\frac{1}{3}$.

Simplify.

Q12

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$$A = \frac{72}{360} \pi(10)^2$$

$$A = 20\pi \text{ cm}^2$$

The sector is $\frac{1}{5}$ of the circle, so multiply the full area by $\frac{1}{5}$.

Simplify.

Q13

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$$L = \frac{1}{2}(2\pi \times 5) = 5\pi$$

Find the curved edge.

$$P = 10 + 5\pi \text{ cm}$$

Add the diameter.

Q14

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$$L = \frac{135}{360}2\pi(8) = 6\pi$$

Calculate its arc length.

$$P = 8 + 8 + 6\pi = 16 + 6\pi \text{ cm}$$

Add both radii.

Q15

$$A = \pi(7^2 - 3^2) = 40\pi \text{ cm}^2$$

Subtract the inner disc from the outer disc.

Q16

$$(a) \quad A = \frac{1}{4}\pi(10^2 - 6^2)$$

Subtract the two quarter-circle areas.

$$A = 16\pi \text{ cm}^2$$

Simplify.

$$(b) \quad L_{\text{outer}} = 5\pi$$

Find the two quarter-circle arcs.

$$L_{\text{inner}} = 3\pi$$

$$\text{straight edges} = 2(10 - 6) = 8$$

There are two radial gaps.

$$P = 8 + 8\pi \text{ cm}$$

Add all four boundary pieces.

Q17

$$A_{\text{sector}} = \frac{1}{4}\pi(6)^2 = 9\pi$$

Find the quarter-circle area.

$$A_{\text{triangle}} = \frac{1}{2}(6)(6) = 18$$

The two radii enclose a right angle.

$$A_{\text{segment}} = 9\pi - 18 \text{ cm}^2$$

Subtract the triangle.

Q18

$$\frac{\theta}{360} 2\pi(12) = 8\pi$$

Use the arc formula.

$$\theta = \frac{8}{24}(360) = 120^\circ$$

Cancel π and solve.

Q19

$$\frac{72}{360} \pi r^2 = 45\pi$$

Use the sector area.

$$r^2 = 225 \Rightarrow r = 15 \text{ cm}$$

Cancel π , rearrange and take the positive root.

Q20

$$\frac{\theta}{360} = \frac{30}{100} = \frac{3}{10}$$

Find its fraction of a full disc.

$$L = \frac{3}{10} 2\pi(10) = 6\pi \text{ cm}$$

Use the same fraction of the circumference.

Q21

$$L = \frac{\theta}{360} 2\pi r$$

Write arc length.

$$\frac{1}{2} r L = \frac{\theta}{360} \pi r^2 = A$$

Multiply by $r/2$ and identify sector area.

Q22

$$(a) \quad A = \frac{1}{2} r L$$

Use the common sector fraction in the arc and area formulas.

$$40 = 5r \Rightarrow r = 8 \text{ cm}$$

Find the radius.

$$(b) \quad P = 10 + 2(8) = 26 \text{ cm}$$

Add the two radii to the arc.

Q23

$$A = \frac{1}{2}(9)L = 4.5L$$

Write both expressions using arc length L .

$$P = L + 18$$

$$4.5L = 3(L + 18)$$

Use the stated numerical relationship.

$$1.5L = 54 \Rightarrow L = 36 \text{ cm}$$

Solve.