

Arcs and sectors

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Circle measures

Circumference is $2\pi r$ and area is πr^2 . A diameter is twice the radius.

A fraction of a circle

A sector of angle θ takes $\theta/360$ of the circumference and area. Arc length is $\frac{\theta}{360} 2\pi r$; sector area is $\frac{\theta}{360} \pi r^2$.

Perimeter

A sector perimeter includes two radii as well as the arc. A compound perimeter follows only the outside boundary.

Exact values

Keep π in an exact answer. For an area made by subtracting a triangle, calculate the triangle from its two radii and included angle.

A calculator may be used.

Calculate from the given information; do not measure the diagrams.

1. Circumference and area

Example Problem 1

★ ★

A circle has diameter 12 cm. Find its exact area.

The idea

1. Find the radius before using the area formula.

Worked Solution

$$r = 6 \text{ cm}$$

Halve the diameter.

$$A = \pi(6)^2 = 36\pi \text{ cm}^2$$

Use radius squared.

1 Circumference and area

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Q1 A circle has radius 7 cm. Find its exact circumference.

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Q2 A circle has diameter 10 cm. Find its exact area.

*

Q3 A circle has circumference 18π cm. Find its exact area.

**

Q4 A circle has area 49π cm². Find its exact circumference.

**

Q5 Two circles have radii 3 cm and 5 cm. Find (a) the ratio of their circumferences; (b) the ratio of their areas.

**

Q6 A wheel has diameter 70 cm. It makes 100 complete revolutions without slipping. Find the exact distance travelled in metres.

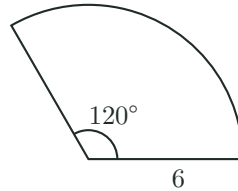
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2. Arc lengths and sector areas

Example Problem 2

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Find the arc length and area of a sector with radius 6 cm and angle 120° .



The idea

1. Use the angle as a fraction of 360° .
2. Apply that fraction separately to circumference and area.

Worked Solution

$$\frac{120}{360} = \frac{1}{3}$$

$$L = \frac{1}{3}(2\pi \times 6) = 4\pi \text{ cm}$$

$$A = \frac{1}{3}\pi(6)^2 = 12\pi \text{ cm}^2$$

The sector is one third of a circle.

Take one third of the circumference.

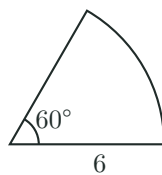
Take one third of the area.

2 Arc lengths and sector areas

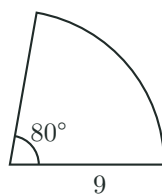
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Q7 Find the exact arc length of a sector with radius 6 cm and angle 60° .

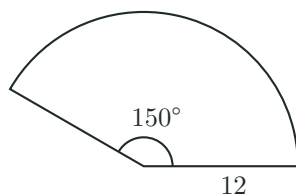
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**Q8** Find the exact arc length of a sector with radius 9 cm and angle 80° .

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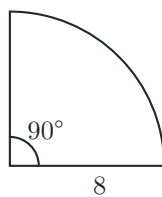
**Q9** Find the exact arc length of a sector with radius 12 cm and angle 150° .

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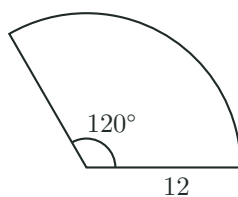
Q10 Find the exact area of a sector with radius 8 cm and angle 90° .

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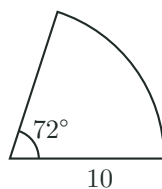
Q11 Find the exact area of a sector with radius 12 cm and angle 120° .

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Q12 Find the exact area of a sector with radius 10 cm and angle 72° .

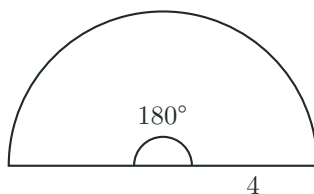
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3. Compound shapes

Example Problem 3

A semicircle has radius 4 cm. Find its exact perimeter.



The idea

1. Trace the full boundary once.
2. The radius is not a boundary edge of a semicircle; the diameter is.

Worked Solution

$$L = \frac{1}{2}(2\pi \times 4) = 4\pi \text{ cm}$$

Find the curved part.

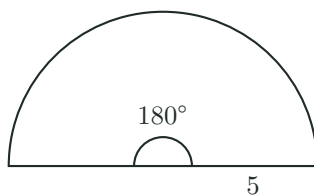
$$P = 8 + 4\pi \text{ cm}$$

Include the straight diameter.

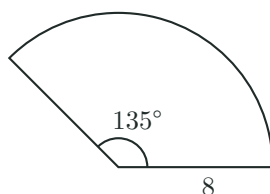
3 Compound shapes

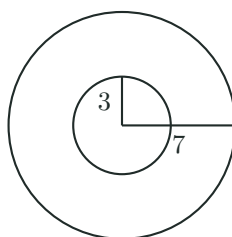
Q13 A semicircle has radius 5 cm. Find its exact perimeter.

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**Q14** A sector has radius 8 cm and angle 135° . Find its exact perimeter.

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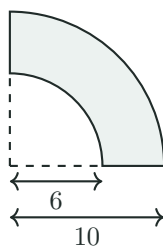
**Q15** A ring has outer radius 7 cm and inner radius 3 cm. Find its exact area.



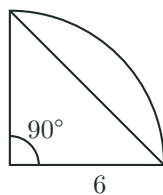
Q16 The diagram shows a quarter of a ring with outer radius 10 cm and inner radius 6 cm.

(a) Find its exact area.

(b) Find its exact perimeter.



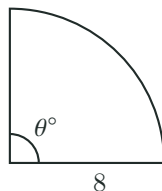
Q17 A sector has radius 6 cm and angle 90° . Its two arc endpoints are joined by a chord. Find the exact area between the chord and the arc.



4. Reverse and algebraic problems

Example Problem 4

A sector has radius 8 cm and arc length 4π cm. Find its angle.



The idea

1. Find what fraction the arc is of a full circumference.
2. Use that fraction of 360° .

Worked Solution

$$\frac{\theta}{360} 2\pi(8) = 4\pi$$

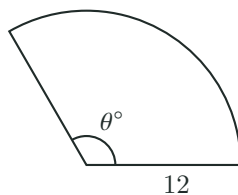
Use the arc length formula.

$$\theta = \frac{4}{16} (360) = 90^\circ$$

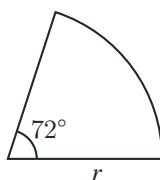
Cancel π and solve.

4 Reverse and algebraic problems

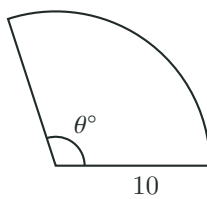
Q18 A sector of radius 12 cm has arc length 8π cm. Find its angle.



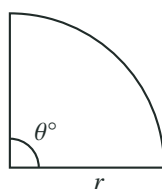
Q19 A sector of angle 72° has area 45π cm². Find its radius.



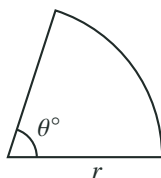
Q20 A sector has radius 10 cm and area $30\pi \text{ cm}^2$. Find its exact arc length.



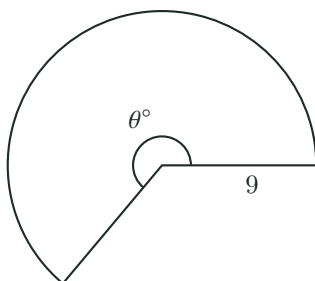
Q21 A sector has radius r cm, arc length L cm and area $A \text{ cm}^2$. Show that $A = \frac{1}{2}rL$.



Q22 A sector has area 40 cm^2 and arc length 10 cm. Find (a) its radius; (b) its perimeter. ***



Q23 A sector has radius 9 cm. The numerical value of its area in square centimetres is three times the numerical value of its perimeter in centimetres. Find its arc length. ***



Solving principles

1. Mark the radius

Check whether a labelled straight length is a radius or diameter.

2. Separate length and area

Circumference scales with r ; area scales with r^2 .

3. Trace the boundary

Count straight edges as well as curved edges, and omit internal joins.

4. Use ratios before calculating

Related sectors often share a radius or angle. Cancelling common factors can simplify a reverse problem.