

GCSE MATHS

Angles, shapes and circles

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Angle facts

Angles on a straight line total 180° ; angles around a point total 360° . Vertically opposite angles are equal.

Parallel lines

Corresponding and alternate angles are equal. Interior angles on the same side of a line crossing two parallel lines total 180° .

Polygons

A triangle has angle sum 180° . An n -sided polygon has interior angle sum $(n - 2)180^\circ$. The exterior angles of a polygon with no inward-pointing corners total 360° .

Circles

A radius joins the centre to the circumference. A diameter passes through the centre. A chord joins two points on the circumference. A tangent touches the circle at one point.

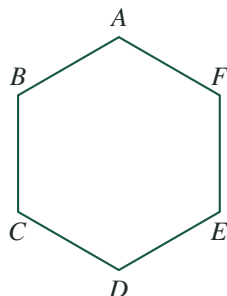
Calculate angles from the given information; do not measure the diagrams.

1. Shapes and symmetry

Example Problem 1

**

Find the smallest positive rotation that maps a regular hexagon onto itself.



The idea

1. Divide a full turn by the number of matching positions.

Worked Solution

$$360^\circ / 6 = 60^\circ$$

There are six matching positions.

1 Shapes and symmetry

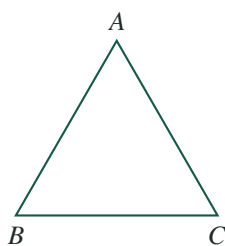
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Q1 How many lines of symmetry does a non-square rectangle have?

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**Q2** State the order of rotational symmetry of an equilateral triangle.

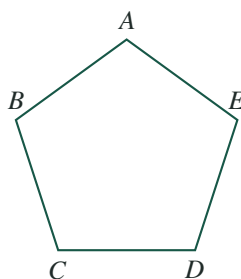
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**Q3** Name a quadrilateral with four equal sides but no right angles.

*

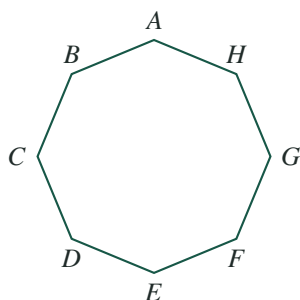
Q4 How many lines of symmetry does a regular pentagon have?

**



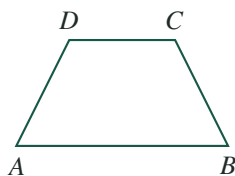
- Q5** A regular polygon has rotational symmetry of order 8. Find its smallest positive angle of rotation.

**



- Q6** A quadrilateral has exactly one pair of parallel sides. What is its name?

**

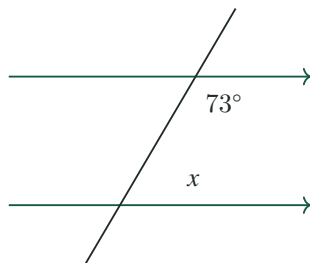


2. Angles and parallel lines

Example Problem 2

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Two co-interior angles between parallel lines are 73° and x . Find x .



The idea

1. Subtract the known angle from 180° .

Worked Solution

$$x + 73^\circ = 180^\circ$$

$$x = 107^\circ$$

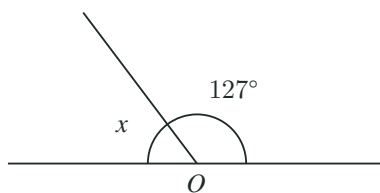
Same-side interior angles sum to 180° .

Subtract 73° .

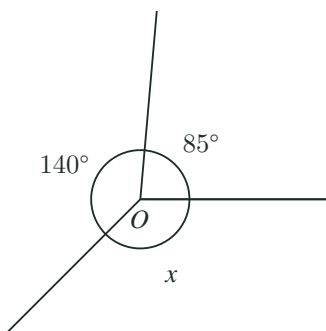
2 Angles and parallel lines

Q7 Find x on the straight line.

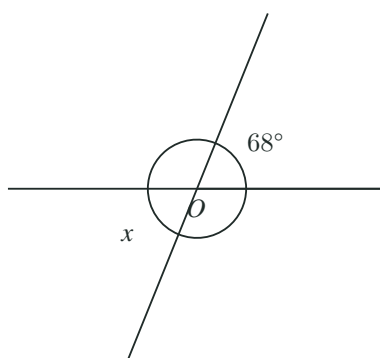
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**Q8** Angles around a point are 85° , 140° and x . Find x .

**

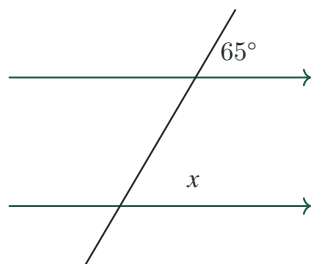
**Q9** Two straight lines cross. One angle is 68° . Find the vertically opposite angle.

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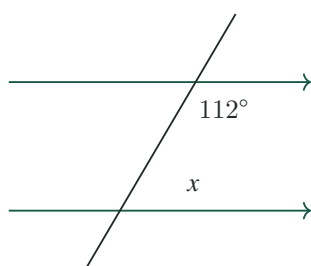
Q10 The horizontal lines are parallel. Find x and give a reason.

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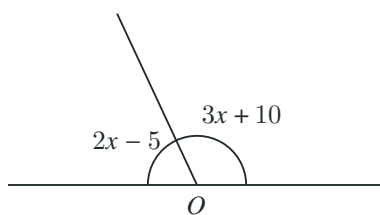


Q11 Two co-interior angles between parallel lines are 112° and x . Find x .

**



Q12 Adjacent angles on a straight line are $(3x + 10)^\circ$ and $(2x - 5)^\circ$. Find x .



3. Triangles and polygons

Example Problem 3

A regular polygon has interior angle 150° . Find its number of sides.

The idea

1. Find its exterior angle, then divide 360° by that angle.

Worked Solution

$$e = 180^\circ - 150^\circ = 30^\circ$$

Adjacent interior and exterior angles form a straight line.

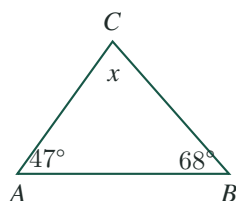
$$n = 360/30 = 12$$

The exterior angles total one turn.

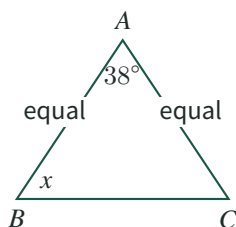
3 Triangles and polygons

Q13 Two angles of a triangle are 47° and 68° . Find the third.

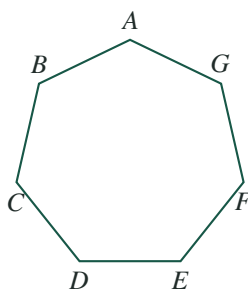
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**Q14** $AB = AC$. Angle BAC is 38° . Find angle ABC .

**

**Q15** Find the interior angle sum of a heptagon.

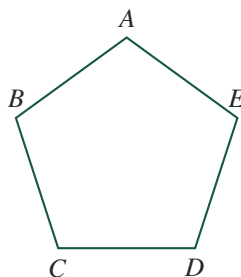
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**Q16** Each exterior angle of a regular polygon is 24° . How many sides does it have?

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Q17 Each interior angle of a regular polygon is 156° . Find the number of sides.

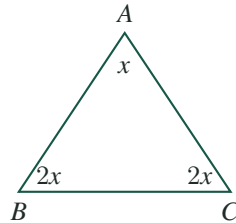
Q18 A pentagon has angles x° , $(x + 10)^\circ$, $(x + 20)^\circ$, $(x + 30)^\circ$ and $(x + 40)^\circ$. Find its largest angle.



4. Circle vocabulary and angle reasoning

Example Problem 4

An isosceles triangle has vertex angle x and two base angles of $2x$ each. Find all three angles.



The idea

1. Use equal base angles and the triangle angle sum.

Worked Solution

$$x + 2x + 2x = 180^\circ$$

$$x = 36^\circ$$

$$36^\circ, 72^\circ, 72^\circ$$

Add all three angles.

Divide by five.

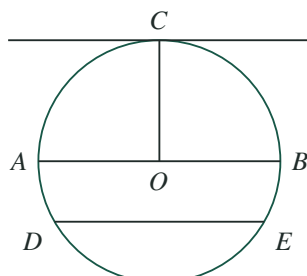
Calculate the two equal base angles.

4 Circle vocabulary and angle reasoning

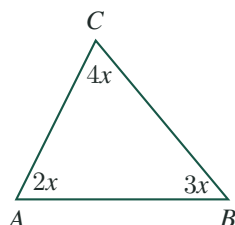
Q19 O is the centre of the circle.

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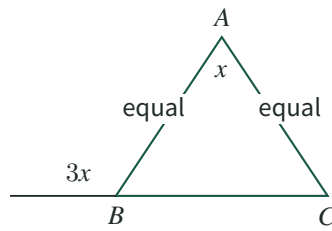
- (a) Which labelled segment is a diameter?
(b) Which labelled segment is a radius other than OA or OB ?
(c) Which labelled segment is a chord that is not a diameter?
(d) What is the mathematical name for the line that touches the circle at C ?



Q20 The angles of a triangle are in the ratio 2:3:4. Find the largest angle.



Q21 An isosceles triangle has $AB = AC$. Its exterior angle at B is three times angle BAC . Find angle BAC .



Solving principles

1. Name the reason

Pair an angle calculation with the fact that allows it.

2. Mark equal angles

Use parallel lines and equal sides to find relationships before calculating.

3. Separate inside and outside

An interior angle and its adjacent exterior angle total 180° .

4. Check the shape

Use the total angle sum to test your result.