

GCSE MATHS

Algebraic fractions

Worked Solutions

Q1

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$$\frac{\frac{12x^3}{18x} = \frac{2x^3}{3x}}{\frac{2x^2}{3}}$$

Divide the numerical coefficients by six.

Cancel a factor of x .

Q2

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$$\frac{(x-3)(x+3)}{x+3}$$

$$x-3$$

Factorise the difference of squares.

Cancel the common factor $x+3$.

Q3

**

$$\frac{(x+2)(x+3)}{x+2}$$

$$x+3$$

Factorise the quadratic.

Cancel $x+2$.

Q4

**

$$\frac{x(x-4)}{(x-4)(x+4)}$$

$$\frac{x}{x+4}$$

$$x \neq 4, -4$$

Factorise top and bottom.

Cancel $x-4$.

Both original denominator factors must be non-zero.

Q5

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$$\frac{(2x+1)(x+3)}{(2x+1)(x-3)}$$

$$\frac{x+3}{x-3}$$

$$x \neq 3, -\frac{1}{2}$$

Factorise both quadratics.

Cancel the common factor $2x+1$.

Retain both original restrictions.

Q6

$$\frac{(3-x)(3+x)}{(x+3)(x-1)}$$

$$\frac{3-x}{x-1}$$

$$x \neq 1, -3$$

Factorise, keeping the order in $3 - x$.Cancel $x + 3$.

Use the original denominator.

Q7

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$$\frac{24x}{36x}$$

$$\frac{2}{3}$$

Multiply numerators and denominators.

Cancel $12x$.**Q8**

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$$\frac{(x-2)(x+2)6x}{3x(x+2)}$$

$$2(x-2)$$

Factorise $x^2 - 4$.Cancel $3x$ and $x + 2$.**Q9**

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$$\frac{x}{5} \times \frac{15}{2x}$$

$$\frac{3}{2}$$

Multiply by the reciprocal of the second fraction.

Cancel x and divide 15 by five.**Q10**

$$\frac{(x-1)(x+1)}{x+2} \times \frac{x+2}{x-1}$$

$$x+1$$

Factorise, then turn the second fraction upside down and multiply.

Cancel the common factors.

Q11

$$\frac{(x-3)(x+3)}{(x+1)(x+3)} \times \frac{x+1}{x-3}$$

$$1$$

Factorise each quadratic.

Every factor cancels.

Q12

$$\frac{(x-2)^2}{(x-2)(x+2)} \times \frac{x+2}{x-2}$$

$$1$$

$$x \neq 2, -2$$

Factorise, then turn the second fraction upside down and multiply.

Cancel the factors.

Original denominators and the divisor must be non-zero.

Q13

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$$\frac{2+5}{x}$$

$$\frac{7}{x}$$

The denominators already match.

Add only the numerators.

Q14

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$$\frac{3(x+1) - 2x}{x(x+1)}$$

$$\frac{x+3}{x(x+1)}$$

Use $x(x+1)$ as the common denominator.

Expand and collect the numerator.

Q15

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$$\frac{2(x+1) + (x-1)}{(x-1)(x+1)}$$

$$\frac{3x+1}{x^2-1}$$

Build a common denominator.

Simplify top and bottom.

Q16

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$$\frac{x-2}{x-2}$$

$$1$$

$$x \neq 2$$

Subtract the numerators.

A non-zero expression divided by itself is one.

Q17

$$\frac{x-2}{(x+2)(x-2)} - \frac{x-2}{(x+2)(x-2)}$$

$$0$$

$$x \neq 2, -2$$

Factorise the denominator and match the fractions.

The fractions cancel, but restrictions remain.

Q18

$$\frac{\frac{(x+1)^2 - (x-1)^2}{x^2 - 1}}{\frac{x^2 + 2x + 1 - (x^2 - 2x + 1)}{x^2 - 1}}$$

$$\frac{4x}{x^2 - 1}$$

Use the product of the denominators.

Bracket the entire subtraction.

Collect like terms.

Q19

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$$15 = 2x$$

$$x = \frac{15}{2}$$

Multiply by $5x$, with x non-zero.

Substitution into the original denominators gives no zero denominator.

Q20

$$2(x+2) = 3(x-1)$$

$$2x + 4 = 3x - 3$$

$$x = 7$$

Multiply by $(x-1)(x+2)$; exclude 1 and -2.

Expand both sides.

Substitution into the original denominators gives no zero denominator.

Q21

$$6(x+1) + 6x = 5x(x+1)$$

$$5x^2 - 7x - 6 = 0$$

$$(5x+3)(x-2) = 0$$

$$x = 2 \text{ or } -\frac{3}{5}$$

Multiply by $6x(x+1)$; exclude 0 and -1.

Expand and rearrange.

Factorise.

Both roots satisfy the restrictions.

Q22

$$x + 2 = x(x - 1)$$

Multiply by $x - 1$; exclude $x = 1$.

$$x^2 - 2x - 2 = 0$$

Expand and rearrange.

$$(x - 1)^2 = 3$$

Complete the square.

$$x = 1 \pm \sqrt{3}$$

Both roots are allowed.

Q23

$$x \neq 2$$

Record the original restriction.

$$(x - 2)(x + 2) = 4(x - 2)$$

Factorise after clearing the denominator.

$$x + 2 = 4$$

Cancel $x - 2$, which is non-zero.

$$x = 2 \text{ is excluded}$$

The value found makes the original denominator zero.

Q24

$$\frac{1}{x} + \frac{1}{x+3} = \frac{1}{2}$$

Add the fractions of a tank filled per hour.

$$2(x + 3) + 2x = x(x + 3)$$

Clear denominators; x is positive.

$$x^2 - x - 6 = 0$$

Expand and rearrange.

$$(x - 3)(x + 2) = 0$$

Factorise.

$$x = 3$$

Reject -2; the slower tap takes $x + 3$ hours.

$$x + 3 = 6 \text{ h}$$