

GCSE MATHS

Algebraic fractions

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Factors, not terms

$\frac{3(x+2)}{6(x+2)} = \frac{1}{2}$ for $x \neq -2$. You can cancel a factor of the whole numerator and denominator.

Restrictions

An original denominator cannot be zero. A restriction remains even if its factor later cancels. For example, $(x-2)/(x-2)$ is undefined at $x=2$ because its denominator is zero.

Division

Divide by a fraction by multiplying by its reciprocal. The fraction you divide by must also be non-zero.

Common denominators

$\frac{1}{x} + \frac{1}{x+1} = \frac{(x+1)+x}{x(x+1)}$. Multiply each numerator by the factor it needs.

1. Cancelling factors

Example Problem 1

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Simplify $\frac{x^2-5x+6}{x^2-9}$ and state the values of x that cannot be used.

The idea

1. Factorise the numerator and denominator completely.
2. Cancel the shared bracket and retain the original restrictions.

Worked Solution

$$\frac{(x-2)(x-3)}{(x-3)(x+3)}$$

$$\frac{x-2}{x+3}$$

$$x \neq 3, -3$$

Factorise both quadratics.

Cancel the whole factor $x-3$.

Neither value that makes an original denominator zero can be used.

1 Cancelling factors

Q1 Simplify $\frac{12x^3}{18x}$, where $x \neq 0$.

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Q2 Simplify $\frac{x^2-9}{x+3}$, where $x \neq -3$.

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Q3 Simplify $\frac{x^2+5x+6}{x+2}$, where $x \neq -2$.

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Q4 Simplify $\frac{x^2-4x}{x^2-16}$. State the values of x that cannot be used.

**

Q5 Simplify $\frac{2x^2+7x+3}{2x^2-5x-3}$. State the values of x that cannot be used.

**

Q6 Simplify $\frac{9-x^2}{x^2+2x-3}$. State the values of x that cannot be used.

2. Multiplying and dividing

Example Problem 2

Simplify $\frac{x^2-16}{x} \div \frac{x+4}{2x}$, where $x \neq 0, -4$.

The idea

1. Invert the entire second fraction.
2. Factorise before cancelling.

Worked Solution

$$\frac{(x-4)(x+4)}{x} \times \frac{2x}{x+4}$$

$$2(x-4)$$

Factorise and replace division by reciprocal multiplication.

Cancel x and $x+4$.

2 Multiplying and dividing

Q7 Simplify $\frac{3x}{4} \times \frac{8}{9x}$, where $x \neq 0$.

**

Q8 Simplify $\frac{x^2-4}{3x} \times \frac{6x}{x+2}$, where $x \neq 0, -2$.

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Q9 Simplify $\frac{x}{5} \div \frac{2x}{15}$, where $x \neq 0$.

**

Q10 Simplify $\frac{x^2-1}{x+2} \div \frac{x-1}{x+2}$, where $x \neq -2, 1$.

Q11 Simplify $\frac{x^2-9}{x^2+4x+3} \times \frac{x+1}{x-3}$, where $x \neq -3, -1, 3$.

Q12 Simplify $\frac{x^2-4x+4}{x^2-4} \div \frac{x-2}{x+2}$ and state the values of x that cannot be used.

3. Adding and subtracting

Example Problem 3

Simplify $\frac{2}{x+1} - \frac{1}{x-2}$, where $x \neq -1, 2$.

The idea

1. Use $(x + 1)(x - 2)$ as the denominator.
2. Put brackets around the second numerator before subtracting.

Worked Solution

$$\frac{2(x-2) - (x+1)}{(x+1)(x-2)}$$

$$\frac{2x - 4 - x - 1}{(x+1)(x-2)}$$

$$\frac{x - 5}{(x+1)(x-2)}$$

Multiply each numerator by its missing factor.

Subtract every term of $x + 1$.

Collect the numerator.

3 Adding and subtracting

Q13 Write $\frac{2}{x} + \frac{5}{x}$ as one fraction, where $x \neq 0$.

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Q14 Write $\frac{3}{x} - \frac{2}{x+1}$ as one simplified fraction, where $x \neq 0, -1$.

**

Q15 Write $\frac{2}{x-1} + \frac{1}{x+1}$ as one fraction, where $x \neq 1, -1$.

**

Q16 Simplify $\frac{x}{x-2} - \frac{2}{x-2}$ and state the value of x that cannot be used.

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Q17 Write $\frac{1}{x+2} - \frac{x-2}{x^2-4}$ as one simplified expression. State the values of x that cannot be used.

Q18 Simplify $\frac{x+1}{x-1} - \frac{x-1}{x+1}$, where $x \neq 1, -1$.

4. Equations and reasoning

Example Problem 4

Solve $\frac{2}{x} + \frac{1}{x+1} = 1$.

The idea

1. Clear both denominators in one multiplication.
2. Solve the quadratic and check the restrictions.

Worked Solution

$$x \neq 0, -1$$

$$2(x+1) + x = x(x+1)$$

$$x^2 - 2x - 2 = 0$$

$$x = 1 \pm \sqrt{3}$$

Record the restrictions.

Multiply every term by $x(x+1)$.

Expand and rearrange.

Complete the square; both roots are allowed.

4 Equations and reasoning

★★★★

Q19 Solve $\frac{3}{x} = \frac{2}{5}$.

★★

Q20 Solve $\frac{2}{x-1} = \frac{3}{x+2}$.

★★★

Q21 Solve $\frac{1}{x} + \frac{1}{x+1} = \frac{5}{6}$.

★★★

Q22 Solve $\frac{x+2}{x-1} = x$.

Q23 Solve $\frac{x^2-4}{x-2} = 4$. Explain why simply solving $x^2 - 4 = 4(x - 2)$ gives a value that is not a solution of the original equation.

Q24 Two taps fill a tank separately in x hours and $(x + 3)$ hours. Together they fill it in 2 hours. How long does the slower tap take to fill the tank on its own?

Solving principles

1. Factorise first

Look for a common factor, a quadratic or a difference of squares before cancelling.

2. Bracket a subtraction

Subtract the entire second numerator. Its signs may change when the bracket is expanded.

3. Clear denominators

Multiply every term in an equation by the common denominator.

4. Check original restrictions

A root that makes an original denominator zero is not a solution.